



## November Monthly Math Challenge High School Level

Instructions: TEAMS coaches submit student answers to the question(s) below using the submission link on the TEAMS website. All submissions must be made during the month of November. Those submissions with correct answers will be entered into a drawing for a \$25 Visa gift card, which will be sent to the student in care of the TEAMS coach.

### Neural Networks

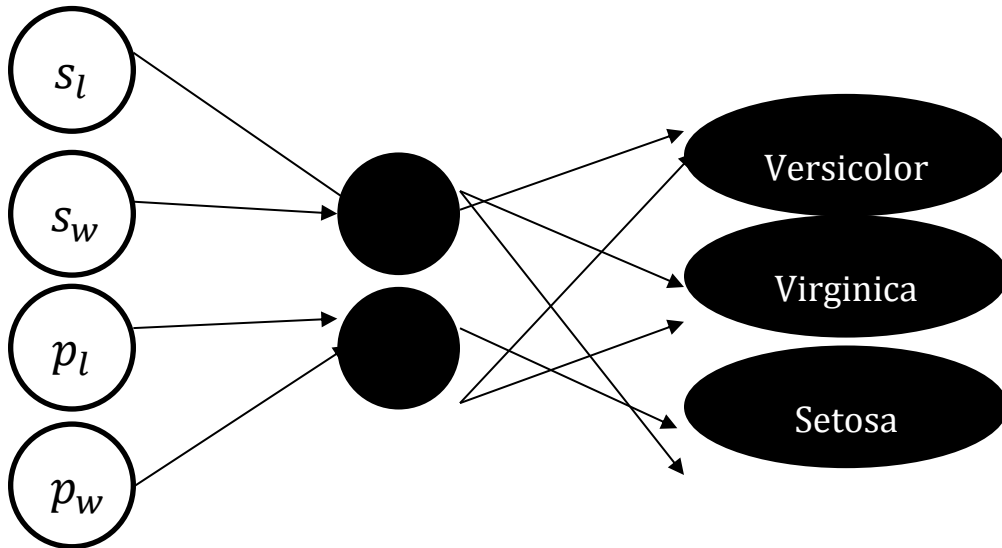
Fisher’s “iris” dataset was introduced in 1936 and serves as a common tool for practitioners to practice or test different classification techniques. It contains 150 observations along four measurements for three species of Iris flowers, with 50 observations each species. For these problems, consider the following subset of the famous “iris” dataset in Table 1.

Table 1. Attributes of flowers and their species, subsetted from the “iris” dataset

Sepal.Length	Sepal.Width	Petal.Length	Petal.Width	Species
$s_l$	$s_w$	$p_l$	$p_w$	$s$
6.7	2.5	5.8	1.8	virginica
6.1	3	4.6	1.4	versicolor
4.9	3	1.4	0.2	setosa
5.4	3.7	1.5	0.2	setosa
5.8	2.7	4.1	1	versicolor
6.1	2.6	5.6	1.4	virginica
4.7	3.2	1.3	0.2	setosa
5.1	3.8	1.6	0.2	setosa
6.3	2.5	5	1.9	virginica
4.4	3.2	1.3	0.2	setosa
5	3.4	1.6	0.4	setosa
6.5	3	5.2	2	virginica
4.6	3.6	1	0.2	setosa
4.8	3	1.4	0.1	setosa
6.3	2.8	5.1	1.5	virginica
6.3	3.3	4.7	1.6	versicolor
6.1	2.9	4.7	1.4	versicolor
5.4	3	4.5	1.5	versicolor
7.7	2.8	6.7	2	virginica
5.2	3.4	1.4	0.2	setosa

Say we construct a neural network to classify the observations based on the four variables (Figure 1) with a generic activation function  $\varphi$ . The neural net is the classifier – a system that categorizes a new observation based on given input data. The predicted class is determined by finding the biggest activation in the final column of neurons.

Figure 1. Neural network for “iris” dataset



Note: Any gray node has the following activation function:

$$\varphi(x) = \frac{1}{1 + e^{-x}}$$

Table 2. Weights on network

The weight is	from	to
0.2	$s_l$	$h_1$
0.4	$s_w$	$h_1$
0.6	$p_l$	$h_2$
0.3	$p_w$	$h_2$
0.5	$h_1$	Versicolor
0.3	$h_1$	Virginica
0.6	$h_1$	Setosa
0.1	$h_2$	Versicolor
0.7	$h_2$	Virginica
0.2	$h_2$	Setosa

**Question 1**

What is the performance of a classifier that randomly guessed the class of each observation?

**Question 2**

Evaluate the performance of the network with all weights equal to 1. Choose the best answer.

- a) the network predicts only a single class
- b) the network cannot predict any class with confidence
- c) the network predicts all observations perfectly, but fails given new data
- d) the network only is accurate with 50 percent of the observations

**Question 3**

What is the activation in Versicolor if all the weights are equal to 1 for an observation with measurements:

Sepal.Length	Sepal.Width	Petal.Length	Petal.Width
$s_l$	$s_w$	$p_l$	$p_w$
6.7	2.5	5.8	1.8