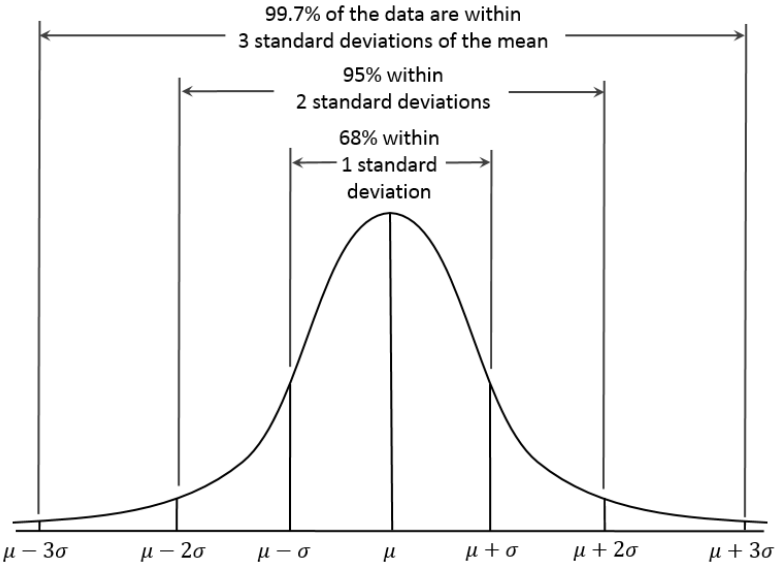
January Monthly Math Challenge Solutions
High School Level

Normal Distribution:

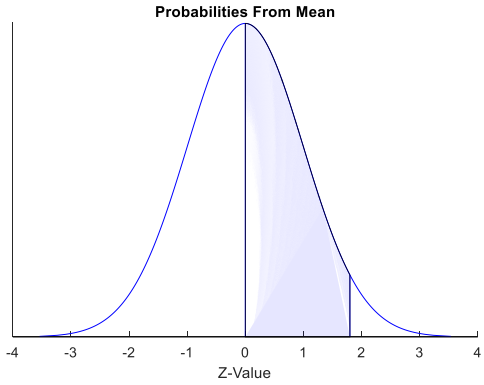
Engineering relies on the analysis of data, and data often presents itself as a normal distribution. Examples of normally distributed data includes exam scores for many students, heights of many people, time to run a mile for a large population of people, and manufacturing information.

The normal distribution has a characteristic look, also known as a bell curve. The normal distribution is symmetrical, centered about its mean. If we know the mean and standard deviation of the data, we know that the area between the mean and +/- one standard deviations equals 68%; the area between the mean and +/- two standard deviations is 95%; the area between the mean and +/- three standard deviations is 99%, etc. Since the curve is symmetrical, 34% of data falls between the mean and +1 SD, and 34% of the data falls between the mean and -1 SD.



Probabilities dealing with full values of standard deviations are often memorized. To find the area beneath the curve between values that are not at exact standard deviation values, the integral of the equation of the normal curve must be determined – or we can use a table of “Z scores”. The relationship of probability, Z-score, SD, and the value in

question is shown in the equation below; the relationship between the Z-score and probability is shown in the Z-table, below.



$$Z = \frac{x - \bar{x}}{SD}$$

Z-scores Probabilities

Z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964

2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

Use the following information and the Z-Table to answer the questions below:

A manufacturer is making widgets, and measuring thousands has shown that the width is normally distributed. The mean is 100 mm and the standard deviation is 15 mm.

Question 1:

What is the probability that any widget is wider than 120 mm?

Question 2:

What is the probability that the width is less than 95 mm?

Solutions:

Question 1: 9.2%

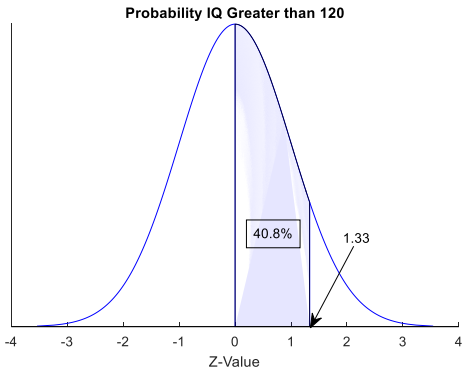
Use the Z-Table to find the probability that the value is greater than 120.

First calculate the Z-score for 120.

$$Z = \frac{120 - 100}{15} = 1.33$$

A Z-score of 1.33 corresponds to a probability of 40.8% as shown in the table. This is found by finding 1.3 on the left most column and using that row to find the value under 0.03 (1.3 + 0.03 = 1.33). 40.8% makes sense because the probability between the mean and +1 SD is 34% and the probability between the mean and +2 SD is about 48%, so a Z-score between 1 and 2 would have a probability between 34% and 48%. When using the Z-table, note that the graphic indicates that the probabilities in the table are from Z = 0.

Using the Z-Table value of 40.8%, the probability of a person having an IQ greater than 120 is 9.2%. This is because the area (probability) that is that to the right of Z = 1.33.



Area under entire curve = 100%. Area left of the mean = 50%. Shaded area = 40.8%

Solution = 100% - 50% - 40.8 % = 9.2%

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Question 2: 37%

The probability that the width is less than 95: although the Z-value of a measurement is less than the mean, because the curve is symmetric, the same Z-table can be used.

$$Z = \frac{|95 - 100|}{15} = 0.33$$

Z = - 0.33, the table shows a probability of about 13.0%. Looking at the graphic, 13.0% is the probability between the mean and 95. Therefore, the probability of an IQ less than 95 is: 50% - 13 %, which equals 37%.