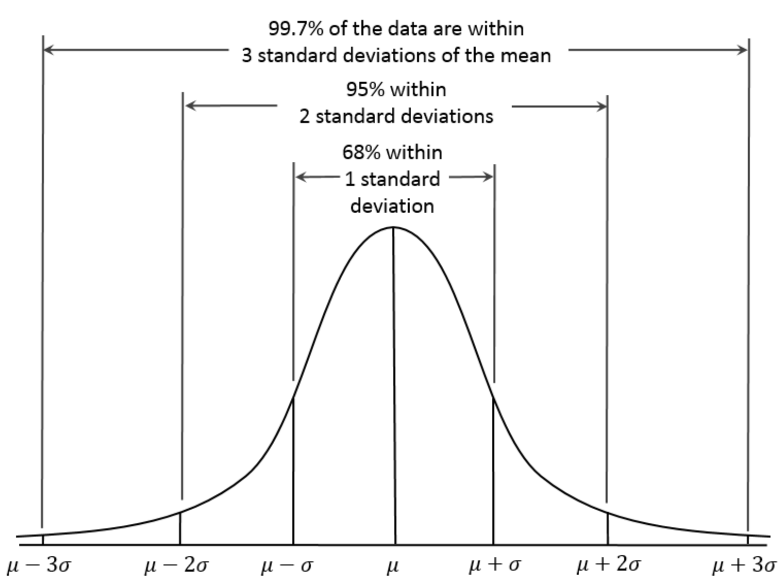
TEAMS Monthly Middle School Math Challenge problem

February 2019

Engineering relies on the analysis of data, and data often presents itself as a normal distribution. Examples of normally distributed data include exam scores for many students, height of many people, time to run a mile for a large population of people, and manufacturing information.

The normal distribution has a characteristic look, also known as a bell curve. The normal distribution is symmetrical about the mean of the data, and if we know the mean and standard deviation of the data, we know that the area between the mean and +/- one standard deviations equals 68%; the area between the mean and +/- two standard deviations is 95%; between the mean and +/- three standard deviations is 99%, etc. Since the curve is symmetrical, 34% of data between the mean and +1 SD, and 34% of the data fall between the mean and -1 SD.

Normal Distribution



Probabilities dealing with full values of standard deviations are often memorized. To find the area beneath the curve between values that are not at exact standard deviation values, a table of “Z scores” is usually used. The relationship of probability, Z-score, SD, and the value in question is shown in the equation below; the relationship between the Z-score and probability is shown in the Z-table, below.

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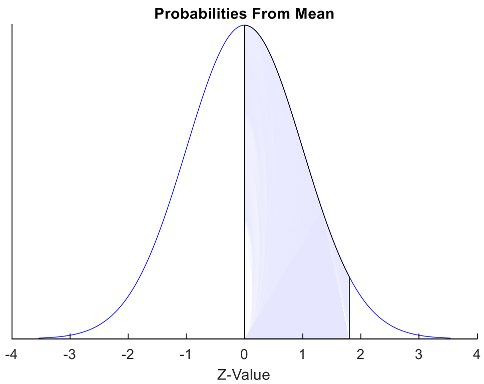

$$Z = \frac{x - \bar{x}}{SD}$$

Note: this value tells us that 46.41% of the data would fall between the mean value and a z value of 1.80, (as shown in the figure)

Z-scores Probabilities

Z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986

3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

Questions:

A manufacturer has developed an edible spoon as an alternative to plastic spoons. They have measured the mass of material used per spoon and found that the mean is 10.0 grams with a standard deviation of 0.8 grams.

1) If we took a sample of 1000 spoons, how many spoons would have a mass between 9.2 and 10.8 grams?
2) With this same sample of 1000 spoons, how many spoons would we expect to have a mass between 10.0 grams and 11.0 grams?

Solution:

1) 680 spoons (any answer between 680 and 683 spoons is acceptable; answer must be a whole number)
2) 394 spoons (answers of 394 or 395 are acceptable; answer much be a whole number)

Details:

1) Students should recognize that the range specified is +/- 1 standard deviation:

   9.2g (10-0.8) to 10.8 g (10+0.8)
   From the reading, 68% of the values fall within this range, giving 680 spoons.

   If we do not recognize this, we can use the Z-table and find

   Z = 10.0-10/0.8 = 1. This gives us a probability of 34.13% from the mean to 10.8. Since the curve is symmetrical, the percentage is 68.26%, or 683 spoons.

2) Use the Z-Table to find the probability that the value is between 10 and 11g.

   First, calculate the Z-score for 11.

$$Z = \frac{11 - 10}{0.8} = 1.25$$

   A Z-score of 1.25 corresponds to a probability of 39.44% as shown in the table, or 394 spoons (we can't have a fraction of a spoon). This is found by finding 1.2 on the left most column and using that row to find the value under 0.05 (1.2 + 0.05 = 1.25). 39.44% makes sense because the probability between the mean and +1 SD is 34% and the probability between the mean and +2 SD is about 48%, so a Z-score between 1 and 2 would have a probability between 34% and 48%. When using the Z-table note that the graphic indicates that the probabilities in the table are from Z = 0.