

December Monthly Math Challenge High School Level

Instructions: TEAMS coaches submit student answers to the question(s) below using the submission link on the TEAMS website. All submissions must be made during the month of December. Those submissions with correct answers will be entered into a drawing for a \$25 Visa gift card, which will be sent to the student in care of the TEAMS coach.

Balance and Hearing in Aquatic Invertebrates

The nervous systems in invertebrates varies widely in order to process and react to the world around them. Invertebrates contain many different organs in order to process sight, sound, and other inputs from their environment. Many aquatic invertebrates, including molluscs, cnidarians (jellyfish), crustaceans, and cephalopods (octopuses), have specialized organs to help them maintain balance and to hear called statocysts. Statocysts are basically hollow balls that are filled with fluid and lined with tiny hairs called setae or cilia which are each connected to a nerve. Within the statocyst is also a small mineralized mass, usually of calcium, called the statolith. The statolith moves through the fluid within the statocyst as the animal moves and changes orientation due to gravity, causing it to brush against the setae. The animal can then make determinations of its orientation, balance, and even acceleration based on the signals received from its setae. This basic system can also provide a cochlea-like mechanism in order to hear.

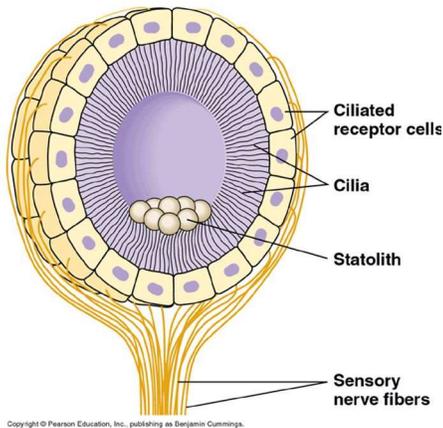


Figure 1: Representation of a statocyst showing cilia and statoliths as well as nerve fibers.

Photo from Pearson Education Inc.

$$\text{Law of sines: } \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\text{Law of cosines: } a^2 = b^2 + c^2 - 2bc \cos A$$

Kinematics equations:

$$v = v_0 + at$$

$$\Delta x = \left(\frac{v + v_0}{2} \right) t$$

$$\Delta x = v_0 t + \frac{1}{2} at^2$$

$$v^2 = v_0^2 + 2a\Delta x$$

Question 1

Let's assume that an octopus is at rest then makes a quick movement downwards and backwards to avoid a predator. We can assume the movement is only in one plane so we can view the statocyst as a circular 2D cross section. After 0.81s into its movement, its nervous system detects the statocyst at a location 24° west of north in our 2D representation.

What was the octopus's acceleration if the statocyst has a circumference of 2.2 cm? Assume the acceleration of the statolith is equivalent to that of the octopus.