TEAMS Monthly High School Math Challenge, January 2017

Directions: Copy and distribute to your students. Coaches may e-mail their student's answers to teams@tsaweb.org --subject line: HS Math Challenge. Make sure to include the student's name, your name, your school, city and state. Correct entries for that month will be placed in a drawing and one name will be randomly drawn on the 2nd Friday of the month following the challenge. The student whose name is drawn will be sent a $25 Visa gift card via their TEAMS coach.

Rules:

1) E-mailed answers must be received by 11:59PM on the last day of the month.
2) All questions must be answered correctly.
3) Answers submitted must be for the current month's posted problem.
4) One entry per student per month allowed.

Innovative Transportation

Any engineering design comes with tradeoffs, and usually, the tradeoffs involve cost. This is true for innovation where we try to more effectively use energy to minimize our effect on the environment. In this case, we will explore a mass transit system to effectively and safely move people along a frequently traveled route.

Analysis has shown that traffic between Roanoke, VA and Washington DC could be dramatically reduced by building a high speed, autonomous rail system. Consider these problems looking at an implementation of an autonomous train.

Given: The speed of light is 3×10^8 m/s.

Simple interest: $I(t) = I_0 rt + I_0$

Compound interest: $I(t) = I_0 \left(1 + \frac{r}{n}\right)^{nt}$

$I(t)$ = investment value after t years
I_0 = initial investment
r = interest rate
n = number of times interest in compounded

February 2017 High School Math Challenge Questions

1. Suppose an autonomous train is traveling toward an intersection; a safety device is sending a signal to the train. The train is moving at 80 mph toward the safety device. At time t_A, the train is 100 meters from the device: a signal is sent from the device to the train. At time t_B, the reflected signal arrives back at the device, having been reflected from the moving train. Find the time from t_A to t_B.

2. If the train in problem #1 decelerates at a rate of 5 m/s^2 until it reaches 70 mph, then decelerates at 2 m/s^2, what is the speed as it reaches the signal?

3. Funding for this system will require investors: Investor #1 will invest $10 million with a compound interest rate of 15%, compounded quarterly. Investor #2 will invest $12 million with a simple interest rate of 12% per annum. Investor #3 will invest $17 million with a compound interest, compounded monthly, of 10%. Finally, Investor #4 will invest $20 million with a simple interest rate of 8%.

    a. Which investor has the largest return after 1 year?
    b. Which investor has the largest return after 10 years?
    c. How many months until the value of the investment of Investor #1 will exceed the value of the investment of Investor #2?

4. How much should be charged per person for the train if the goal is to charge the total amount spent on fuel for the car trips that will be replaced? Assume the average cost of fuel is $2.25/gallon, the average car gets 22 mpg and has 1.5 passengers and the average trip is 240 miles.