

TEAMS Monthly Middle School Math Challenge, February 2017

Directions: Copy and distribute to your students. Coaches may e-mail their student's answers to teams@tsaweb.org --subject line: MS Math Challenge. Make sure to include the student's name, your name, your school, city and state. Correct entries for that month will be placed in a drawing and one name will be randomly drawn on the 2nd Friday of the month following the challenge. The student whose name is drawn will be sent a \$25 Visa gift card via their TEAMS coach.

Rules:

- 1) E-mailed answers must be received by 11:59PM on the last day of the month.
- 2) All parts to the monthly question must be answered correctly. If two questions are posed, both must be answered correctly.
- 3) Answers submitted must be for the current month's posted problem.
- 4) One entry per student per month allowed.

Innovative Transportation

Any engineering design comes with tradeoffs, and usually, the tradeoffs involve cost. This is true for innovation where we try to more effectively use energy to minimize our effect on the environment. In this case, we will explore a mass transit system to effectively and safely move people along a frequently traveled route.

Analysis has shown that traffic between Roanoke, VA and Washington DC could be dramatically reduced by building a high speed, autonomous rail system. Consider these problems looking at an implementation of an autonomous train.

Given: The speed of light is 3×10^8 m/s.

| | |
|---|---|
| Simple interest: $I(t) = I_0rt + I_0$ | Equations of motion: $d = v_i \cdot t + \frac{1}{2} \cdot a \cdot t^2 \quad v_f^2 = v_i^2 + 2 \cdot a \cdot d$ $v_f = v_i + a \cdot t \quad d = \frac{v_i + v_f}{2} \cdot t$ |
| Compound interest: $I(t) = I_0 \left(1 + \frac{r}{n}\right)^{nt}$ | I(t) = investment value after t years I ₀ = initial investment r = interest rate n = number of times interest is compounded |

February 2017 Middle School Math Challenge

1. Suppose an autonomous train is traveling toward an intersection; a safety device is sending a signal to the train. The train is moving at 80 mph toward the safety device. At time t_1 , the train is 100 meters from the device: a signal is sent from the device to the train. At time t_2 , the reflected signal arrives back at the device after being reflected from the moving train. Find the time from t_1 to t_2 .
2. If the train decelerates at a rate of 5 m/s^2 until it reaches 70 mph, then decelerates at 2 m/s^2 , what is the speed as it reaches the signal?
3. Funding for this system will require investors: Investor #1 will invest \$10 million with a compound interest rate of 15%, compounded quarterly. Investor #2 will invest \$12 million with a simple interest rate of 12% per annum. Investor #3 will invest \$17 million with a compound interest, compounded monthly, of 10%. Finally, Investor #4 will invest \$20 million with a simple interest rate of 8%.
 - a) Which investor has the largest return after 1 year?
 - b) Which investor has the largest return after 10 years?
4. How much should be charged per person for the train if the goal is to charge the total amount spent on fuel for the car trips that will be replaced? Assume the average cost of fuel is \$2.25/gallon, the average car gets 22 mpg and has 1.5 passengers and the average trip is 240 miles.