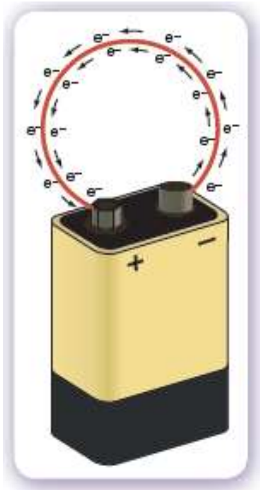
December math problems – middle school

Background:

Voltage (V) Electrical potential (units: Volts, or V)
Current (I) The flow of electricity (units: Amps, or A)
Resistance (R) Characteristic of electrical components that resists current flow (units: ohms, or Ω)
Ohm's Law I = VxR: the voltage across a component equals its resistance multiplied by the current through it.
Series Resistors in series require all current flowing through one resistor to also flow through the next resistor, and so on.
Parallel Resistors in parallel are connected to each other at both ends. They have the same voltage across them, but any current flowing into the parallel combination of these resistors splits among them and joins back together after passing through.
Kirchhoff's Voltage Law (**KVL**) The sum of all voltages around a path in a circuit must equal zero.
Kirchhoff's Current Law (**KCL**) The sum of all currents entering a node must be zero; in other words, any current entering a node must also leave the node.
Node A junction of electronic components.

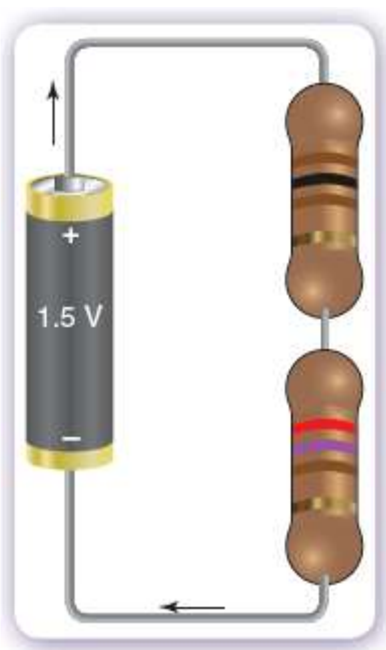
We have all seen batteries in flashlights, game controllers and many other devices. The side of a AA battery says "1.5V", meaning it can supply 1.5 Volts. A 9V battery supplies – you guessed it: 9 Volts.

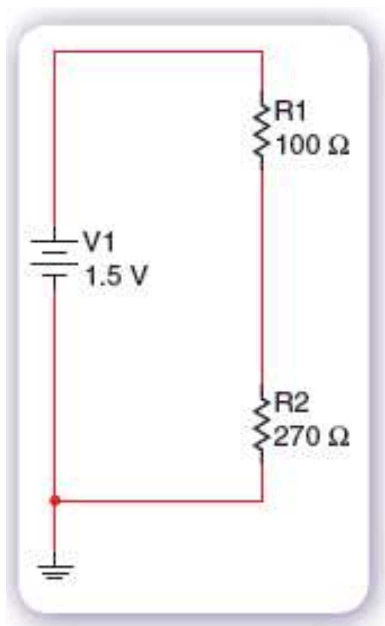


This 9V battery in a package is ready to create current, but it needs to be connected to a complete circuit to generate current. Once we create a path from the positive to the negative terminal, the battery can use it's voltage to generate a current through the circuit. If we complete the circuit, or create a path from one battery terminal back to the other, we let charge flow out of one terminal and back into the other. This flow of charge is current. As we've discussed, we need to be sure we don't ask the battery to send an infinite amount of current through our circuit. When we add resistance to the circuit, we limit the amount of current that can flow in the circuit. In reality, anything that allows electricity to flow through it has some resistance, but for some things, resistance can be very, very low. Resistance (as the name implies) describes a device that resists the flow of current. The higher the resistance, the harder it is for current to flow.

A simple circuit can be a battery (or voltage source) attached to one or more resistors (a device that controls the amount of current that can be generated) as shown: the 1.5V battery sends current through these two resistors.

We can calculate the current sent through the resistance (note – in this case, these resistors are in series and we can add the resistances together – more on this later). The picture on the left is represented by sybols on the right – both of these show the same circuit.





Using Ohm's Law, we can calculate the current, I:

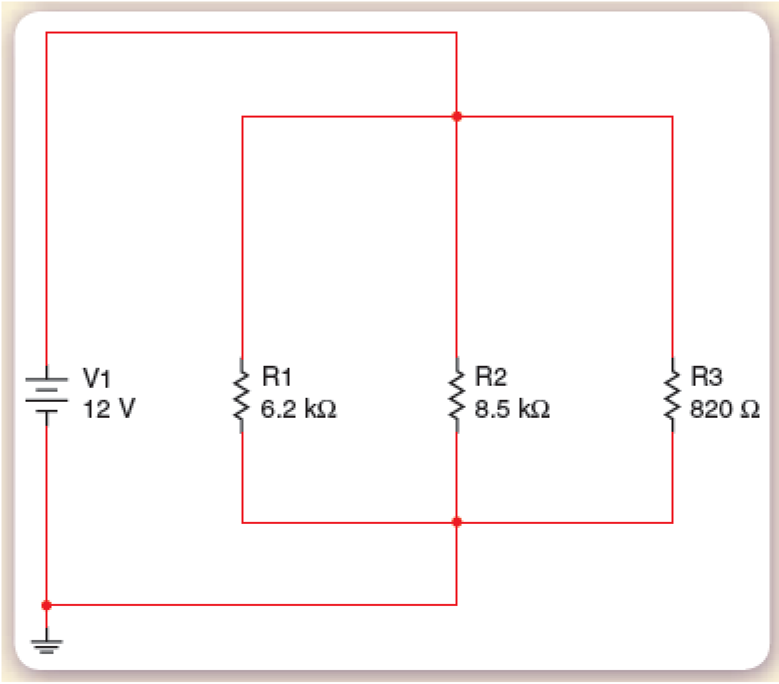
I = V/R = 1/5V / Rtotal

where Rtotal = 270Ω + 100Ω = 370Ω

Therefore, I = 1.5V / 370Ω = 0.00405 A or 4.05mA

The resistors in our example are in series: all of the current through the first resistor must also go through the second. When resistors are in series, we can add their resistances:

Rtotal = R1 + R2 = R3 + …



Resistors that are precisely next to each other – where the top of one is connected to the top of the other and the bottom is connected to the bottom (where the top are connected to the same node and the bottom are connected to the same node) are in parallel. In this case, the current from the 12V battery must split between three different paths.

To calculate the current flow through each of these resistors, we first need to find the total equivalent resistance, Rtotal.

$$Rtotal = \frac{1}{{}^{1}/_{R1} + {}^{1}/_{R2} + {}^{1}/_{R3} + \cdots}$$

In this case, Rtotal = $\frac{1}{{}^{1}/_{6.2k} + {}^{1}/_{8.5k} + {}^{1}/_{820}}$ = 667Ω

Then, we can find the total current flowing out of the battery using Ohm's Law:

I = V/R = 12/667 = 0.018A - This is the total current, and it is split 3 ways – some through each resistor.

We can find the current through each resistor:

The current through R1 can be found since we know the Voltage across it (12V) and its R value.

I = V/R = 12/6.2k = 0.00194A

likewise, through R2: I = V/R = 12/8.5k = 0.00141A

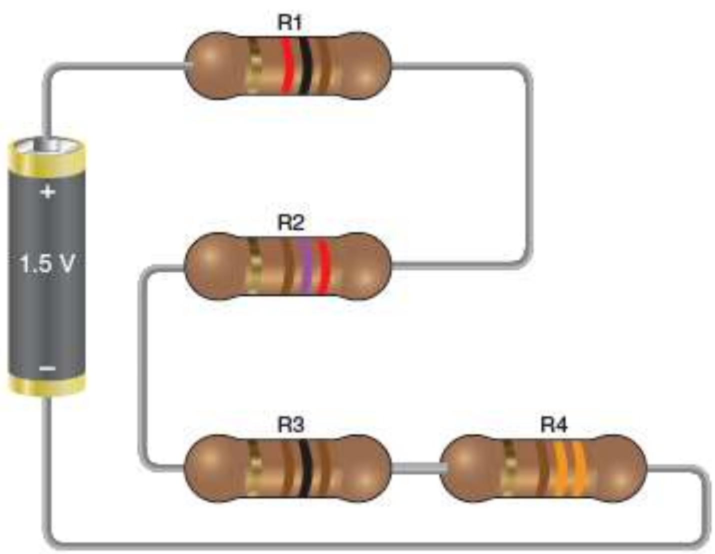
and I = V/R = 12/820 = 0.0146A

Interestingly, if we add these up, we should get the total current:
.00194 + .00141 + .0146 = 0.0180A ☺

Questions:

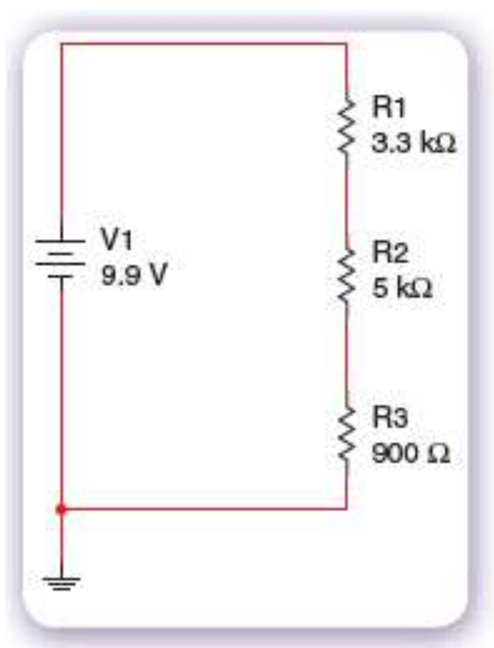
1a. Is the circuit below series or parallel? Why?

b. Find the equivalent resistance in and current through the circuits shown:



R1 = 1000Ω, R2 = 270Ω,

R3 = 100Ω and R4 = 330Ω



1a. They are in series, because all current must flow through R1, then R2, etc.

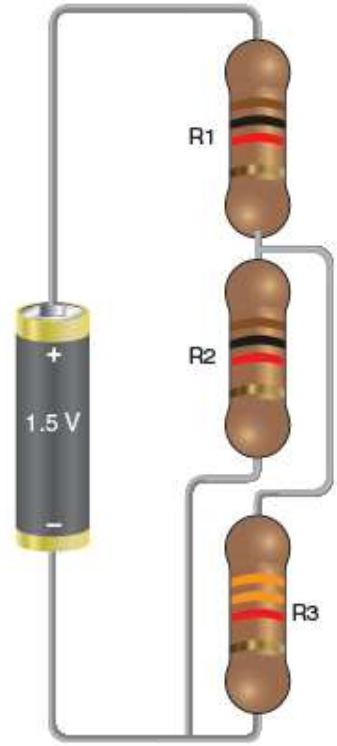
1b The equivalent resistance is

R1+R2+R3+R4 =
1000 + 270 + 100 + 330 =
1700Ω

Bottom circuit –

3.3k + 5k + 900 = 9200Ω

2a. Are R2 and R3 in series or parallel? Why?
2b. Find the total current from the battery in terms of V, R1, R2 and R3.



2a. R2 and R3 are parallel – the top share a node, the bottom share a node

2b. The total R is: $R1 + \frac{1}{1/R2 + 1/R2}$

The current is I = V/R

Therefore, the current is: $V / (R1 + \frac{1}{1/R2 + 1/R2})$

Note – students may look up the value of the resistors based on the color stripes. If they do so, the answer would be:

$1.5 / (1000 + \frac{1}{1/1000 + 1/3300}) = 848\ \mu A$ (or 84.8 mA or 8.48×10^{-4}A)

The equation or the final value are acceptable answers.